Practice Test

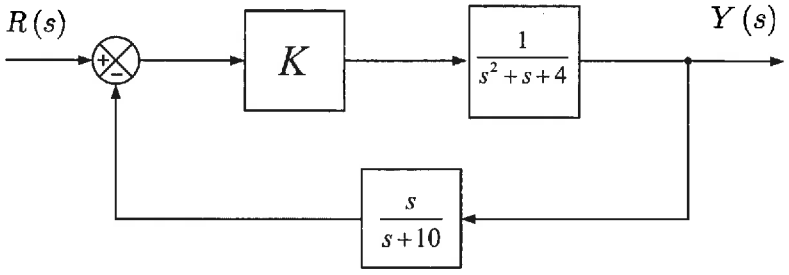
Midterm #2

ECE 317: Feedback and Control

- Closed book and closed notes, except as described below.
- One only ($8\frac{1}{2}" \times 11"$) page of handwritten notes is permitted. (Written on both sides is OK).
- Calculators are OK
- Scrap paper is not to be used. Show all work on the exam paper.

Student name: SOLUTION

Problem 1.
Find the range of K for stability.



Your answer: $K > -10.36$

$$\frac{Y}{R} = \frac{\frac{K}{s^2+s+4}}{1 + \frac{K}{s^2+s+4}\,\frac{s}{s+10}} = \frac{K(s+10)}{(s+10)(s^2+s+4)+Ks} = \frac{K(s+10)}{s^3+s^2+4s+10s^2+10s+40+Ks}$$

$$= \frac{K(s+10)}{s^3 + 11s^2 + (14+K)s + 40}$$

s^3	1	$14+K$
s^2	11	40
s^1	x	
s^0	40	

$$x = \frac{40 - 11(14+K)}{-11} = 14 + K - \frac{40}{11}$$

FOR STABILITY $\Rightarrow$ $14 + K - \frac{40}{11} > 0$

$\Rightarrow$ $K > \frac{40}{11} - 14$

$\Rightarrow$ $K > -10.36$

Problem 2.
Find the forced sinusoidal response of the system $G(s)$ to input $r(t)$ where:

$$G(s) = \frac{10}{s+2}, \qquad r(t) = 5\cos(6t - 45^\circ)$$

Your answer: $7.9\cos(6t - 116.57^\circ)$

$\omega = 6$

$$G(j\omega)\Big|_{\omega=6} = \frac{10}{j\omega+2}\Bigg|_{\omega=6} = \frac{10}{2+j6} = \frac{5}{1+j3}$$

$$\Rightarrow |G(j6)| = \left|\frac{5}{1+j3}\right| = \frac{5}{\sqrt{1+9}} = \frac{5}{\sqrt{10}} = 1.58$$

$$\angle G(j6) = \angle\frac{5}{1+j3} = \angle 5 - \angle 1+j3 = 0 - \tan^{-1}(3) = -71.57^\circ$$

$$\Rightarrow \text{FORCED SINUSOIDAL RESPONSE} = 5\,|G(j6)|\cos\left(6t - 45^\circ + \angle G(j6)\right)$$

$$= 5 \times 1.58 \cos(6t - 45^\circ - 71.57^\circ)$$

$$= 7.9\cos(6t - 116.57^\circ)$$

Problem 3:

Given the following loop gain:

$$T(s) = \frac{A\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)^2}$$

where

$$A = 16$$
$$\omega_z = 1000 \text{ rds/s}$$
$$\omega_p = 50 \text{ rds/s}$$

Using asymptotic approximations only,

a) Sketch the Bode magnitude and phase plots (on the next page). Be sure to label all break frequencies, slopes of sloping line, gains of sloping lines and gain and phase levels on zero slope lines.

b) Using your plots determine the phase margin and associated crossover frequency.

c) It was found that the -180° phase crossover frequency is 320 rds/s, using your plots determine the gain margin (in dB).

d) Determine whether the closed loop system is stable. Justify your answer.

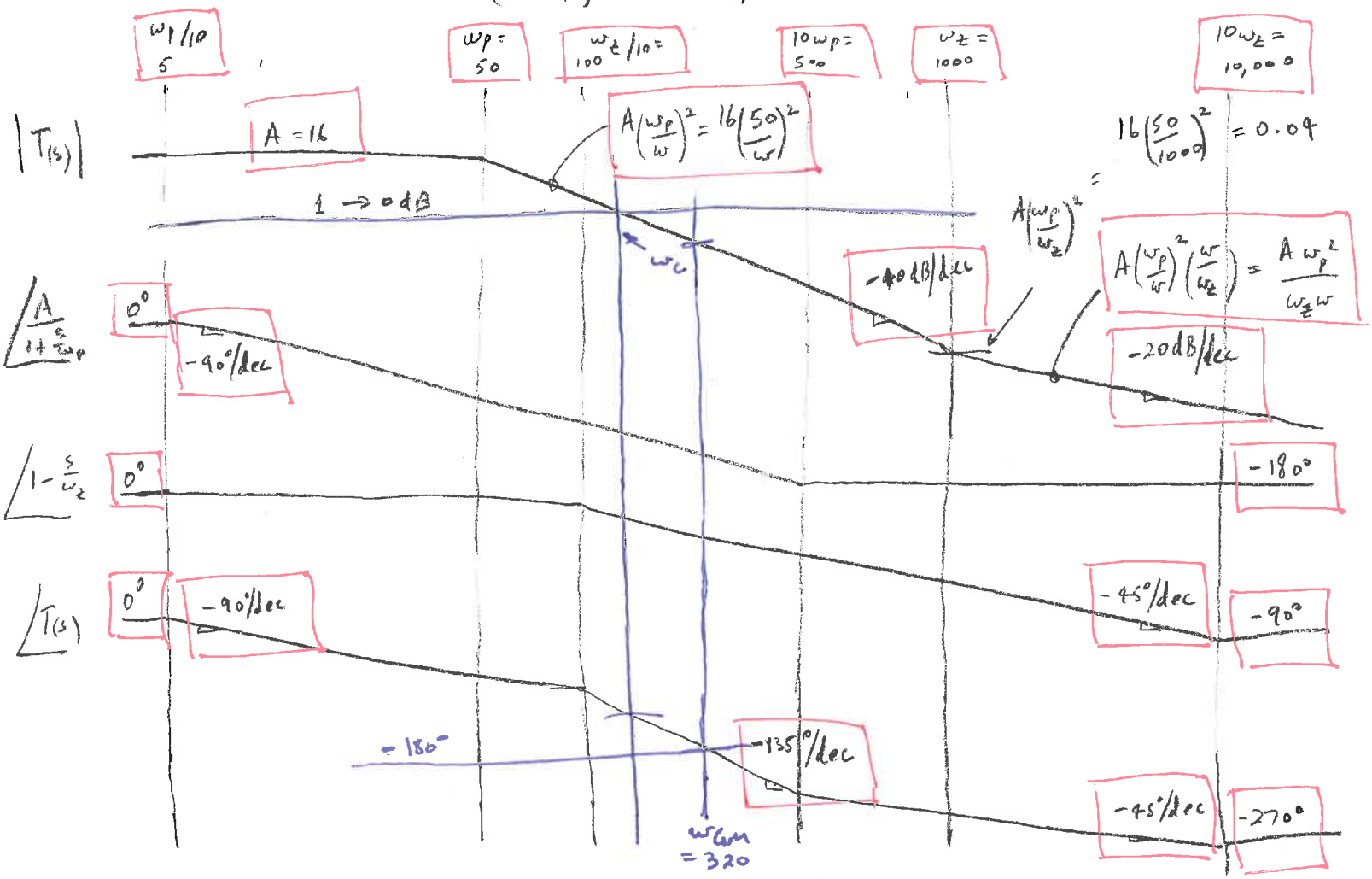
PROBLEM 3:

$$T(s) = \frac{A\left(1 - \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_p}\right)^2}$$

$A = 16$

$\omega_z = 1000$ rds/s

$\omega_p = 50$ rds/s



a) SKETCH SHOWN ABOVE, RED BOXES INDICATE REQUIRED ANNOTATIONS

b) UNITY GAIN CROSSOVER FREQUENCY @ $\omega = \omega_c \Rightarrow A\left(\frac{\omega_p}{\omega_c}\right)^2 = 1 \Rightarrow \omega_c = \omega_p\sqrt{A} = 50\sqrt{16} = 200$ rds/s

PHASE @ $\omega_c = \underbrace{-2\,\text{atan}\left(\frac{\omega_c}{\omega_p}\right)}_{\text{DUE TO DOUBLE POLE}} - \underbrace{\text{atan}\left(\frac{\omega_c}{\omega_z}\right)}_{\text{DUE TO RHP ZERO}}$

$= -2\,\text{atan}\left(\frac{200}{50}\right) - \text{atan}\left(\frac{200}{1000}\right)$

$= -2\,\text{atan}(4) - \text{atan}(0.2)$

$= -163^\circ$

$\Rightarrow$ PHASE MARGIN (PM) $= 180^\circ - 163^\circ = 17^\circ$

c) GAIN @ $\omega = \omega_{GM} = A\left(\frac{\omega_p}{\omega_{GM}}\right)^2 = 16\left(\frac{50}{320}\right)^2 = 0.3902$

$GM = -20\log_{10}(0.3902) = 7.96$ dB

d) STABLE. SINCE **PM** $= 17^\circ > 0$