FALL 2018

Midterm #1

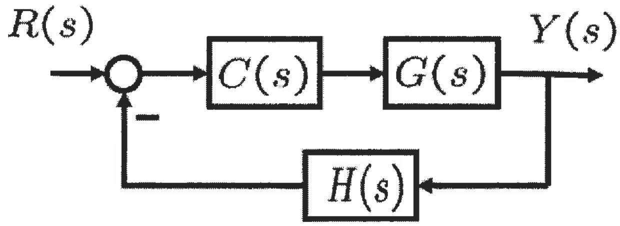
ECE 317: Feedback and Control

- Closed book and closed notes, except as described below.
- One only ($8\frac{1}{2}'' \times 11''$) page of notes is permitted. (Written on both sides is OK).
- No calculators can be used.
- Scrap paper is not to be used. Show all work on the exam paper.

Student name: SOLUTION

(10 pts) **Problem 1.**

Given the system below where the plant $G(s) = \frac{1}{s-1}$, the feedback gain, $H(s) = \frac{1}{2}$, and $C(s) = k$ (a constant).



(2pts) a) Is the (open loop) plant stable? Why?

$G(s) = \frac{1}{s-1}$ POLE @ $s=1$ RHP $\Rightarrow$ UNSTABLE

(3pts) b) Find the closed loop transfer function, $\frac{Y(s)}{R(s)}$.

$$\frac{Y}{R} = \frac{CG}{1+CGH} = \frac{k\frac{1}{s-1}}{1+\frac{k}{2}\frac{1}{s-1}} = \frac{k}{s+\left(\frac{k}{2}-1\right)}$$

(2pts) c) Find the range of k for which the closed loop system is stable.

From (b) REQUIRE $\frac{k}{2} \geq 1 \Rightarrow k \geq 2$

(3pts) d) Under the condition of a very high loop gain, what is the closed loop gain very well approximated by?

$$\frac{Y}{R} = \frac{1}{H}\frac{CGH}{1+CGH} \approx \frac{1}{H}$$ WHEN LOOP GAIN IS

LARGE i.e. $|CGH| \gg 1$ $\Rightarrow \frac{Y}{R} = \frac{1}{H} = \frac{1}{\frac{1}{2}} = 2.$

(10 pts) **Problem 2.**

A second order system has pole locations at $-3 \pm 4j$ determine the following:

(2 pts) a) the damping ratio, ς

$$(s+3+4j)(s+3-4j) = s^2 + 6s + \underbrace{9+16}_{25}$$

COMPARE WITH $s^2 + 2\zeta\omega_n s + \omega_n^2$ $\Rightarrow$ $\omega_n^2 = 25$ $\Rightarrow$ $\omega_n = \sqrt{25} = 5$

$\Rightarrow 2\zeta\omega_n = 6 \Rightarrow \zeta = \frac{6}{2\omega_n} = \frac{3}{5} = 0.6$

(2 pts) b) the undamped natural frequency, ω_n

FROM (a)

$\omega_n = 5$

OR

$\zeta = \cos\theta = \frac{A}{H} = \frac{3}{5} = 0.6$.

(2 pts) c) the damped natural frequency, ω_d (i.e. the observed oscillation frequency)

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 5\sqrt{1-(0.6)^2} = 5\sqrt{1-0.36} = 5\sqrt{0.64} = 5 \times 0.8 = 4 \text{ rad/s.}$$

(2 pts) d) this system (which does not feature any zeros) is excited by a unit step input, determine the time it would take for the output to settle to within a $\pm 2\%$ band of the final steady state value.

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.6 \times 5} = \frac{4}{3} \text{ s}$$

(2 pts) e) how would you characterize the transient response: undamped, underdamped, critically damped or overdamped? Justify your answer.

$\zeta = 0.6$, $0 < \zeta = 0.6 < 1$ $\Rightarrow$ UNDERDAMPED

(10 pts) **Problem 3.**

(2 pts) a) Write the definition of asymptotic stability.

ANY INITIAL CONDITIONS GENERATE AN OUTPUT $y(t)$ CONVERGING TO ZERO

(2 pts) b) Write the definition of BIBO stability.

ANY BOUNDED INPUT GENERATES A BOUNDED OUTPUT

c) Determine if $G(s)$ in the table is stable, marginally stable or unstable. Justify your answers.

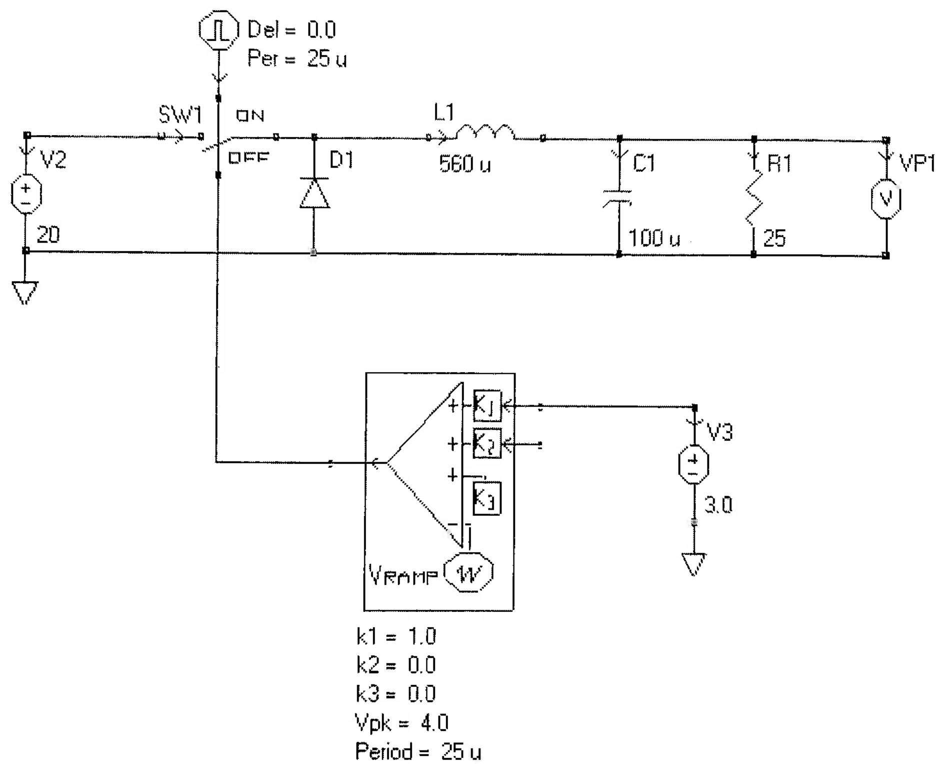
	$G(s)$	Stable/ marginally stable/ unstable
(1 pt)	$\frac{s-2}{s(s+1)(s+2)^2}$	POLES @ $s=0, -1, -2, -2$ ⇒ MARGINALLY STABLE (SIMPLE IMAG. AXIS ROOT)
(1 pt)	$\frac{s-2}{s(s+1)(s+2)}$	POLES @ $s=0, -1, -2$ ⇒ MARGINALLY STABLE (SIMPLE IMAG AXIS ROOT)
(2 pts)	$\frac{s-2}{s(s+1)(s^2+2)}$	POLES @ $s=0, -1, \pm j2$ ⇒ MARGINALLY STABLE (SIMPLE IMAG. AXIS ROOTS)
(2 pts)	$\frac{s-2}{s(s+1)(s^2+2)^2}$	POLES @ $s=0, -1, \pm j2, \pm j2$ ⇒ UNSTABLE (MULTIPLE IMAG AXIS ROOTS.)

SOLUTION

ECE317 Bonus Question

(This question is not mandatory)

Provide answers to the two questions below concerning the circuit shown by the PECS schematic below.



1) What is the switching frequency?

THE PERIOD T_s IS SPECIFIED BY THE CLOCK AND THE ALSO BY THE PWM. $T_s = 25\times10^{-6}$ $\Rightarrow$ $f = \frac{1}{T_s} = \frac{1}{25\times10^{-6}} = 40\ kHz$

2) What is the average voltage at the output as measured by VP1?

THE PWM SWITCHES STATE WHEN THE 4 VOLT RAMP REACHES 3 VOLTS (AS SPECIFIED BY V3). THIS RESULTS IN A DUTY RATIO D OF $D = \frac{3}{4} = 0.75$

FOR BUCK CONVERTER $VP1 = D\ V2$

$= 0.75 \times 20$

$= 15\ V$