# Solution:

# ECE317
# HW #9

**Problem 1:**
Determine the type of the following unity-feedback systems for which the forward-path transfer functions are given:

**(a)** $G(s) = \frac{K}{(1+s)(1+10s)(1+20s)}$

**(b)** $G(s) = \frac{10e^{-0.2s}}{(1+s)(1+10s)(1+20s)}$

**(c)** $G(s) = \frac{10(s+1)}{s(s+5)(s+6)}$

**(d)** $G(s) = \frac{100(s-1)}{s^2(s+5)(s+6)^2}$

**(e)** $G(s) = \frac{10(s+1)}{s^3(s^2+5s+5)}$

**(f)** $G(s) = \frac{100}{s^3(s+2)^2}$

**(g)** $G(s) = \frac{5(s+2)}{s^2(s+4)}$

**(h)** $G(s) = \frac{8(s+1)}{(s^2+2s+3)(s+1)}$

**Solution:**

**(a)** Type 0 **(b)** Type 0 **(c)** Type 1 **(d)** Type 2 **(e)** Type 3 **(f)** Type 3

**(g)** type 2 **(h)** type 0

**Problem 2:**
Determine the step, ramp and parabolic error constants of the following unity-feedback control systems. The forward-path transfer functions are given by:

(a) $G(s) = \dfrac{1000}{(1+0.1s)(1+10s)}$ (b) $G(s) = \dfrac{100}{s(s^2+10s+100)}$

(c) $G(s) = \dfrac{K}{s(1+0.1s)(1+0.5s)}$ (d) $G(s) = \dfrac{100}{s^2(s^2+10s+100)}$

(e) $G(s) = \dfrac{1000}{s(s+10)(s+100)}$ (f) $G(s) = \dfrac{K(1+2s)(1+4s)}{s^2(s^2+s+1)}$

**Solution:**

**(a)** $K_p = \lim_{s\to 0} G(s) = 1000$ $K_v = \lim_{s\to 0} sG(s) = 0$ $K_a = \lim_{s\to 0} s^2G(s) = 0$

**(b)** $K_p = \lim_{s\to 0} G(s) = \infty$ $K_v = \lim_{s\to 0} sG(s) = 1$ $K_a = \lim_{s\to 0} s^2G(s) = 0$

**(c)** $K_p = \lim_{s\to 0} G(s) = \infty$ $K_v = \lim_{s\to 0} sG(s) = K$ $K_a = \lim_{s\to 0} s^2G(s) = 0$

**(d)** $K_p = \lim_{s\to 0} G(s) = \infty$ $K_v = \lim_{s\to 0} sG(s) = \infty$ $K_a = \lim_{s\to 0} s^2G(s) = 1$

**(e)** $K_p = \lim_{s\to 0} G(s) = \infty$ $K_v = \lim_{s\to 0} sG(s) = 1$ $K_a = \lim_{s\to 0} s^2G(s) = 0$

**(f)** $K_p = \lim_{s\to 0} G(s) = \infty$ $K_v = \lim_{s\to 0} sG(s) = \infty$ $K_a = \lim_{s\to 0} s^2G(s) = K$

**Problem 3:**

For the unity-feedback control systems described in Problem 2, determine the steady-state error for a unit-step, $u_s(t)$, unit-ramp, $tu_s(t)$, and parabolic input $\left(\frac{t^2}{2}\right) u_s(t)$. Check the stability of the system before applying the final-value theorem.

**(a)** $G(s) = \dfrac{1000}{(1 + 0.1s)(1 + 10s)}$

**(b)** $G(s) = \dfrac{100}{s(s^2 + 10s + 100)}$

**(c)** $G(s) = \dfrac{K}{s(1 + 0.1s)(1 + 0.5s)}$

**(d)** $G(s) = \dfrac{100}{s^2(s^2 + 10s + 100)}$

**(e)** $G(s) = \dfrac{1000}{s(s + 10)(s + 100)}$

**(f)** $G(s) = \dfrac{K(1 + 2s)(1 + 4s)}{s^2(s^2 + s + 1)}$

**Solution:**

**(a)**

| Input | Error Constants | Steady-state Error |
|---|---|---|
| $u_s(t)$ | $K_p = 1000$ | 1/1001 |
| $tu_s(t)$ | $K_v = 0$ | $\propto$ |
| $t^2 u_s(t)/2$ | $K_a = 0$ | $\propto$ |

**(b)**

| Input | Error Constants | Steady-state Error |
|---|---|---|
| $u_s(t)$ | $K_p = \infty$ | 0 |
| $tu_s(t)$ | $K_v = 1$ | 1 |
| $t^2 u_s(t)/2$ | $K_a = 0$ | $\propto$ |

**(c)**

| Input | Error Constants | Steady-state Error |
|---|---|---|
| $u_s(t)$ | $K_p = \infty$ | 0 |
| $tu_s(t)$ | $K_v = K$ | $1/K$ |
| $t^2u_s(t)/2$ | $K_a = 0$ | $\propto$ |

The above results are valid if the value of *K* corresponds to a stable closed-loop system.

**(d)** The closed-loop system is unstable. It is meaningless to conduct a steady-state error analysis.

**(e)**

| Input | Error Constants | Steady-state Error |
|---|---|---|
| $u_s(t)$ | $K_p = \infty$ | 0 |
| $tu_s(t)$ | $K_v = 1$ | 1 |
| $t^2u_s(t)/2$ | $K_a = 0$ | $\propto$ |

**(f)**

| Input | Error Constants | Steady-state Error |
|---|---|---|
| $u_s(t)$ | $K_p = \infty$ | 0 |
| $tu_s(t)$ | $K_v = \infty$ | 0 |
| $t^2u_s(t)/2$ | $K_a = K$ | $1/K$ |

The closed-loop system is stable for all positive values of *K*. Thus the above results are valid.

**Problem 4:**

The following transfer functions are given for a single-loop non-unity-feedback control system. Determine the steady errors for a unit-step, $u_s(t)$, unit-ramp, $tu_s(t)$, and parabolic input, $\left(\frac{t^2}{2}\right) u_s(t)$.

(a) $G(s) = \frac{1}{(s^2+s+2)}$ $\quad H(s) = \frac{1}{(s+1)}$

(b) $G(s) = \frac{1}{s(s+5)}$ $\quad H(s) = 5$

(c) $G(s) = \frac{1}{s^2(s+10)}$ $\quad H(s) = \frac{s+1}{s+5}$

(d) $G(s) = \frac{1}{s^2(s+12)}$ $\quad H(s) = 5(s+2)$

**Solution:**

**(a)** $K_H = H(0) = 1$ $\quad M(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{s+1}{s^3+2s^2+3s+3}$

$$a_0 = 3, \quad a_1 = 3, \quad a_2 = 2, \quad b_0 = 1, \quad b_1 = 1.$$

**Unit-step Input:**

$$e_{ss} = \frac{1}{K_H}\left(1 - \frac{b_0 K_H}{a_0}\right) = \frac{2}{3}$$

**Unit-ramp input:**

$$a_0 - b_0 K_H = 3 - 1 = 2 \neq 0. \text{ Thus } e_{ss} = \infty.$$

**Unit-parabolic Input:**

$$a_0 - b_0 K_H = 2 \neq 0 \text{ and } a_1 - b_1 K_H = 1 \neq 0. \text{ Thus } e_{ss} = \infty.$$

**(b)** $K_H = H(0) = 5$

$$M(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{1}{s^2+5s+5} \qquad a_0 = 5, \quad a_1 = 5, \quad b_0 = 1, \quad b_1 = 0.$$

**Unit-step Input:**

$$e_{ss} = \frac{1}{K_H}\left(1 - \frac{b_0 K_H}{a_0}\right) = \frac{1}{5}\left(1 - \frac{5}{5}\right) = 0$$

**Unit-ramp Input:**

$$i = 0: \quad a_0 - b_0 K_H = 0 \qquad i = 1: \quad a_1 - b_1 K_H = 5 \neq 0$$

$$e_{ss} = \frac{a_1 - b_1 K_H}{a_0 K_H} = \frac{5}{25} = \frac{1}{5}$$

**Unit-parabolic Input:**

$$e_{ss} = \infty$$

**(c)** $K_H = H(0) = 1/5$

$$M(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{s+5}{s^4+15s^3+50s^2+s+1} \qquad \text{The system is stable.}$$

$$a_0 = 1, \quad a_1 = 1, \quad a_2 = 50, \quad a_3 = 15, \quad b_0 = 5, \quad b_1 = 1$$

**Unit-step Input:**

$$e_{ss} = \frac{1}{K_H}\left(1 - \frac{b_0 K_H}{a_0}\right) = 5\left(1 - \frac{5/5}{1}\right) = 0$$

**Unit-ramp Input:**

$$i = 0: \quad a_0 - b_0 K_H = 0 \qquad i = 1: \quad a_1 - b_1 K_H = 4/5 \neq 0$$

$$e_{ss} = \frac{a_1 - b_1 K_H}{a_0 K_H} = \frac{1 - 1/5}{1/5} = 4$$

**Unit-parabolic Input:**

$$e_{ss} = \infty$$

**(d)** $K_H = H(0) = 10$

$$M(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{1}{s^3 + 12s^2 + 5s + 10}$$ The system is stable.

$$a_0 = 10, \quad a_1 = 5, \quad a_2 = 12, \quad b_0 = 1, \quad b_1 = 0, \quad b_2 = 0$$

**Unit-step Input:**

$$e_{ss} = \frac{1}{K_H}\left(1 - \frac{b_0 K_H}{a_0}\right) = \frac{1}{10}\left(1 - \frac{10}{10}\right) = 0$$

**Unit-ramp Input:**

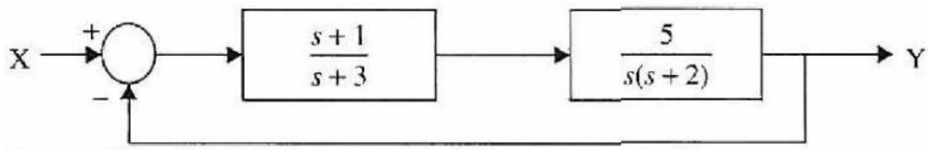
$$i = 0: \quad a_0 - b_0 K_H = 0 \quad i = 1: \quad a_1 - b_1 K_H = 5 \neq 0$$

$$e_{ss} = \frac{a_1 - b_1 K_H}{a_0 K_H} = \frac{5}{100} = 0.05$$

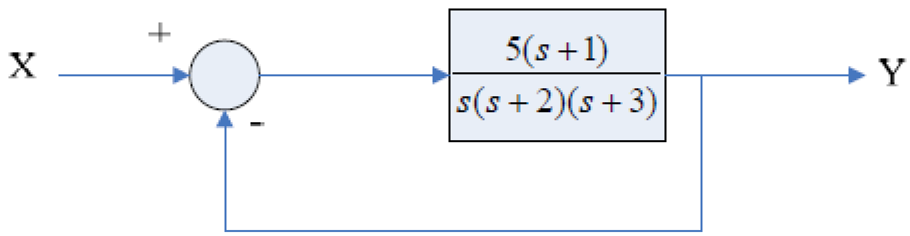
**Unit-parabolic Input:**

$$e_{ss} = \infty$$

**Problem 5:**
Find the position, velocity and acceleration constants for the system given below.




**Solution:**




$$G(s) = \frac{5(s+1)}{s(s+2)(s+3)}$$

a) Position error: $K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{5(s+1)}{s(s+2)(s+3)} = \infty$

b) Velocity error: $K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} \frac{5(s+1)}{(s+2)(s+3)} = \frac{5}{6}$

c) Acceleration error: $K_a = \lim_{s\to \infty} s^2 G(s) = \lim_{s\to \infty} \frac{5s(s+1)}{(s+2)(s+3)} = 0$

**Problem 6:**
For the system of Problem 5, find the steady-state error for (a) a unit-step, $u_s(t)$, (b) a unit-ramp, $tu_s(t)$, and (c) a unit parabolic input, $\left(\frac{t^2}{2}\right) u_s(t)$.

**Solution:**

a) Steady state error for unit step input:

$$e_{ss} = \frac{1}{1+K_p}$$

Referring to the result of Problem 4, $K_p = \infty \implies e_{ss} = 0$

b) Steady state error for ramp input:

$$e_{ss} = \frac{1}{K_v}$$

Referring to the result of Problem 4, $K_v = \frac{5}{6} \implies e_{ss} = \frac{6}{5}$

c) Steady state error for parabolic input:

$$e_{ss} = \frac{1}{K_a}$$

Referring to the result of Problem 4, $K_a = 0 \implies e_{ss} = \infty$