ECE317
HW #6

Problem 1:

Find the forced sinusoidal response to the indicated signals:

a) $F(s) = \dfrac{1}{s+8}, \quad r(t) = 5\cos 5t$

b) $F(s) = \dfrac{s^2}{s+4}, \quad r(t) = 100\cos(4t + 40^\circ)$

c) $F(s) = \dfrac{10}{s^2 + 2s + 10}, \quad r(t) = 4\cos(5t - 70^\circ)$

d) $F(s) = \dfrac{s}{(s+4)(s+8)}, \quad r(t) = 10\cos(4t + 120^\circ)$

e) $F(s) = \dfrac{10}{(s+6)^2}, \quad r(t) = 5\cos 6t$

Problem 2:

Find analytical expressions for the magnitude and phase response for each $G(s)$ below:

a) $G(s) = \dfrac{1}{s+1}$

b) $G(s) = \dfrac{2}{s^2 + 3s + 2}$

Solutions

Problem 1:

(a) $F(j\omega) = \frac{1}{j\omega+8}$ $A(\omega) = \frac{1}{\sqrt{\omega^2+64}}$ $\phi(\omega) = -\tan^{-1}\frac{\omega}{8}$

$\omega = 5$ $A(\omega) = 0.106$ $\phi(\omega) = -\tan^{-1}\frac{5}{8} = -32^\circ$

$y(t) = 5(.106)\cos(5t-32^\circ) = \boxed{0.53\cos(5t-32^\circ)}$

(b) $F(j\omega) = \frac{(j\omega)^2}{j\omega+4}$ $A(\omega) = \frac{\omega^2}{\sqrt{\omega^2+16}}$ $\phi(\omega) = 180^\circ - \tan^{-1}\frac{\omega}{4}$

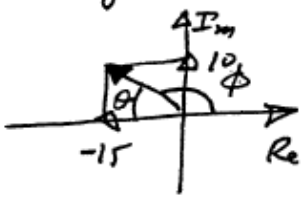
$\omega = 4$ $A(\omega) = \frac{4^2}{\sqrt{4^2+16}} = 2.83$ $\phi(\omega) = 180^\circ - \tan^{-1}1 = 135^\circ$

$y(t) = 100(2.83)\cos(4t+40^\circ+135^\circ) = \boxed{283\cos(4t+175^\circ)}$

(c) $F(j\omega) = \frac{10}{(j\omega)^2+2j\omega+10} = \frac{10}{10-\omega^2+j(2\omega)} \overset{\omega=5}{=} \frac{10}{-15+j10}$

$A(\omega) = \frac{10}{\sqrt{(-15)^2+10^2}}$

$= 0.555$



$\phi(\omega) = -[180^\circ - \theta] = -[180^\circ - \tan^{-1}\frac{10}{15}] = -146.3^\circ$

$y(t) = 4(0.555)\cos(5t-70^\circ-146.3^\circ) = 2.22\cos(5t-216.3^\circ)$

$= \boxed{2.22\cos(5t+143.7^\circ)}$

(d) $F(j\omega) = \frac{j\omega}{(j\omega+4)(j\omega+8)}$ $A(\omega) = \frac{\omega}{\sqrt{\omega^2+16}\sqrt{\omega^2+64}}$

$\phi(\omega) = 90^\circ - \tan^{-1}\frac{\omega}{4} - \tan^{-1}\frac{\omega}{8}$ $\omega = 4$

$A(\omega) = \frac{4}{\sqrt{16+16}\sqrt{16+64}} = 0.079$ $\phi(\omega) = 90^\circ - \tan^{-1}1 - \tan^{-1}0.5$

$= 18.4^\circ$

$y(t) = 10(0.079)\cos(4t+120^\circ+18.4^\circ)$

$= \boxed{0.79\cos(4t+138.4^\circ)}$

e)

$F(j\omega) = \frac{10}{(j\omega+6)^2}$ $A(\omega) = \frac{10}{(\sqrt{\omega^2+36})^2} = \frac{10}{\omega^2+36}$

$\phi(\omega) = -2\tan^{-1}\frac{\omega}{6}$ $\omega=6$ $A(\omega) = \frac{10}{36+36} = 0.139$

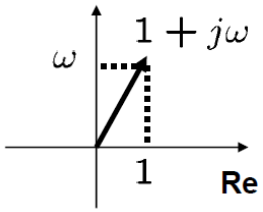
$\phi(\omega) = -2\tan^{-1}1 = -90^\circ$

$y(t) = 5(0.139)\cos(6t-90^\circ) = \boxed{0.7\cos(6t-90^\circ)}$

Problem 2:

a)

$$G(s) = \frac{1}{s+1} \Rightarrow G(j\omega) = \frac{1}{j\omega+1}$$



$$\Rightarrow \begin{cases} |G(j\omega)| = \dfrac{1}{\sqrt{1+\omega^2}} \\ \angle G(j\omega) = \angle(1) - \angle(j\omega+1) = -\tan^{-1}\omega \end{cases}$$

b)

$$G(s) = \frac{2}{s^2+3s+2}$$

$$\Rightarrow G(j\omega) = \frac{2}{(j\omega)^2+3(j\omega)+2} = \frac{2}{2-\omega^2+j\cdot 3\omega}$$

$$\Rightarrow \begin{cases} |G(j\omega)| = \dfrac{2}{\sqrt{(2-\omega^2)^2+9\omega^2}} \\ \angle G(j\omega) = \angle(2) - \angle(2-\omega^2+j\cdot 3\omega) \\ \qquad\quad = -\tan^{-1}\dfrac{3\omega}{2-\omega^2} \end{cases}$$