SOLUTION

ECE317
HW #4

1) Why are pole locations of a transfer function important?

A TRANSFER FUNCTION'S POLE LOCATIONS DETERMINE ITS STABILITY

2) How do you determine if a transfer function is stable, marginally stable or unstable?

A STABLE TRANSFER FUNCTION HAS POLES IN THE OPEN LHP

A MARGINALLY STABLE TRANSFER FUNCTION HAS AT LEAST ONE SIMPLE POLE ON THE $j\omega$ AXIS BUT NO MULTIPLE POLES ON THE $j\omega$ AXIS.

AN UNSTABLE TRANSFER FUNCTION IS ONE THAT IS NEITHER STABLE NOR MARGINALLY STABLE.

3) Determine the stability of following transfer functions:

$G(s)$	Stable/ marginally stable /unstable
$\frac{5(s+2)}{(s+1)(s^2+s+1)}$	? POLES: $\left\{-1, -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right\}$ $\Rightarrow$ STABLE
$\frac{5(-s+2)}{(s+1)(s^2+s+1)}$	? POLES: $\left\{-1, -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right\}$ $\Rightarrow$ STABLE
$\frac{5}{(s-2)(s^2+3)}$	? POLES: $\left\{2, \pm j\sqrt{3}\right\}$ $\Rightarrow$ UNSTABLE
$\frac{s^2+3}{(s+1)(s^2-s+1)}$	? POLES: $\left\{-1, \frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right.$ $\Rightarrow$ UNSTABLE
$\frac{1}{(s+1)(s^2+1)^2}$	? POLES: $\left\{-1, \pm j, \pm j\right\}$ $\Rightarrow$ UNSTABLE
$\frac{1}{(s^2-1)(s+1)}$	? POLES: $\left\{\pm 1, -1\right\}$ $\Rightarrow$ UNSTABLE
$\frac{20}{(s+1)(s+2)(s+3)}$	? POLES: $\left\{-1, -2, -3\right\}$ $\Rightarrow$ STABLE
$\frac{20(s+1)}{(s-1)(s^2+2s+3)}$	? POLES: $\left\{1, -1 \pm j\sqrt{2}\right\}$ $\Rightarrow$ UNSTABLE
$\frac{10(s-1)e^{-s}}{(s+5)(s^2+3)}$	? POLES: $\left\{-5, \pm j\sqrt{3}\right\}$ $\Rightarrow$ MARGINALLY STABLE
$\frac{1}{(s+5)(s^2+2)^2}$	? POLES: $\left\{-5, \pm j\sqrt{2}, \pm j\sqrt{2}\right\}$ $\Rightarrow$ UNSTABLE

4) The relationship between three variables, v, v_g and d is given by the following:

$$v = \frac{1}{1-d} v_g$$

As this relationship is nonlinear we are interested in finding a linearized model by considering small-signal deviations around a steady state operating point. To this end we consider each of the variables to consist of a steady state value plus a small deviation from the steady state such that

$$v = V + \hat{v} \quad \text{where} \quad \hat{v} << V$$

$$v_g = V_g + \hat{v}_g \quad \text{where} \quad \hat{v}_g << V_g$$

$$d = D + \hat{d} \quad \text{where} \quad \hat{d} << D$$

Capitalized terms represent steady state values and terms with a caret '^' indicate a small deviation from the steady state. Find the transfer functions: $\frac{\hat{v}}{\hat{d}}$ and $\frac{\hat{v}}{\hat{v}_g}$.

SOLUTION:

$$v = \frac{1}{1-d} v_g \Rightarrow v(1-d) = v_g$$

$$\Rightarrow (V+\hat{v})(1-(D+\hat{d})) = V_g + \hat{v}_g$$

$$\Rightarrow V(1-D) - V\hat{d} + (1-D)\hat{v} - \hat{v}\hat{d} = V_g + \hat{v}_g$$

EQUATING FIRST ORDER TERMS $\Rightarrow$

$$-V\hat{d} + (1-D)\hat{v} = \hat{v}_g$$

$$\Rightarrow \boxed{\frac{\hat{v}}{\hat{d}} = \frac{V}{1-D}} \quad (\hat{v}_g = 0)$$

AND

$$\boxed{\frac{\hat{v}}{\hat{v}_g} = \frac{1}{1-D}} \quad (\hat{d} = 0)$$

ALSO

NOTE FROM THE STEADY STATE MODEL: $V(1-D) = V_g$

$$\Rightarrow V = \frac{V_g}{1-D}$$

SO THAT

$$\frac{\hat{v}}{\hat{d}} = \frac{V_g}{(1-D)^2}$$

ALTERNATIVE SOLUTION USING TAYLOR SERIES EXPANSION

$$v = f(d, v_g) \quad \leftarrow \text{NON LINEAR FUNCTION}$$

TAYLOR EXPANSION (LINEAR TERMS ONLY):

$$v = f(d, v_g)\Big|_{\substack{d=D \\ v_g=V_g}} + \frac{\partial f(d, v_g)}{\partial d}\Big|_{\substack{d=D \\ v_g=V_g}}(d - D) + \frac{\partial f(d, v_g)}{\partial v_g}\Big|_{\substack{d=D \\ v_g=V_g}}(v_g - V_g)$$

denote $\hat{d} = d - D$

and $\hat{v}_g = v_g - V_g$

ALSO LET $v = V + \hat{v}$ $\left(\Rightarrow \hat{v} = v - V\right)$

FOR $v = \frac{1}{1-d} v_g$

$$\Rightarrow V + \hat{v} = \frac{1}{1-D} V_g + \frac{\partial\left(\frac{v_g}{1-d}\right)}{\partial d}\Bigg|_{\substack{d=D \\ v_g=V_g}} \hat{d} + \frac{\partial\left(\frac{v_g}{1-d}\right)}{\partial v_g}\Bigg|_{\substack{d=D \\ v_g=V_g}} \hat{v}_g$$

$$= \frac{1}{1-D} V_g + \frac{v_g}{(1-d)^2}\Bigg|_{\substack{d=D \\ v_g=V_g}} \hat{d} + \frac{1}{1-d}\Bigg|_{\substack{d=D \\ v_g=V_g}} \hat{v}_g$$

$$\underbrace{V}_{\substack{\text{DC} \\ \text{TERM}}} + \underbrace{\hat{v}}_{\substack{\text{1ST ORDER} \\ \text{TERM}}} = \underbrace{\frac{1}{1-D} V_g}_{\substack{\text{0 ORDER TERM} \\ \text{(DC TERM)}}} + \underbrace{\frac{V_g}{(1-D)^2} \hat{d} + \frac{1}{1-D} \hat{v}_g}_{\text{1ST ORDER TERMS}}$$

$$\Rightarrow \quad V = \frac{1}{1-D} V_g$$

AND $\frac{\hat{v}}{\hat{d}}\Big|_{\hat{v}_g=0} = \frac{V_g}{(1-D)^2}$ AND $\frac{\hat{v}}{\hat{v}_g}\Big|_{\hat{d}=0} = \frac{1}{1-D}$