ECE317
HW #3 Solutions

Probs. 2a, 4, a, b, d and f of problems 8, 17 and 18, 20a , 23a and c, 24

Solutions:

Prob. 2a:

a. $C(s) = \frac{5}{s(s+5)} = \frac{1}{s} - \frac{1}{s+5}$. Therefore, $c(t) = 1 - e^{-5t}$.

Also, $T = \frac{1}{5}$, $T_r = \frac{2.2}{a} = \frac{2.2}{5} = 0.44$, $T_s = \frac{4}{a} = \frac{4}{5} = 0.8$.

Prob. 4:

Using voltage division, $\frac{V_C(s)}{V_i(s)} = \frac{1/RC}{s + \frac{1}{RC}} = \frac{1}{s+1}$. Since $V_i(s) = \frac{5}{s}$

$$V_C(s) = \frac{5}{s}\left(\frac{1}{s+1}\right) = \frac{5}{s} - \frac{5}{s+1}.$$

Therefore: $v_c(t) = 5 - 5e^{-t}$.

Also, $T = \frac{1}{1} = 1\sec; T_r = \frac{2.2}{1} = 2.2\sec; T_s = \frac{4}{1} = 4\sec$.

Prob. 8:

a. Pole: -2; $c(t) = A + Be^{-2t}$; first-order response.

b. Poles: -3, -6; $c(t) = A + Be^{-3t} + Ce^{-6t}$; overdamped response.

d. Poles: $(-3+j3\sqrt{15})$, $(-3-j3\sqrt{15})$; $c(t) = A + Be^{-3t}\cos(3\sqrt{15}\,t + \phi)$; underdamped.

f. Poles: -10, -10; Zero: -5; $c(t) = A + Be^{-10t} + Cte^{-10t}$; critically damped.

Prob. 17:

a.

$$C(s) = \frac{2}{s(s+2)}$$

$$C(s) = \frac{1}{s} - \frac{1}{s+2}$$

$$c(t) = 1 - e^{-2t}$$

b.

$$C(s)=\frac{5}{s(s+3)(s+6)}$$

$$C(s)=\frac{5}{18}\frac{1}{s}-\frac{5}{9}\frac{1}{s+3}+\frac{5}{18}\frac{1}{s+6}$$

$$c(t)=\frac{5}{18}-\frac{5}{9}e^{-3t}+\frac{5}{18}e^{-6t}$$

d.

$$C(s)=\frac{20}{s(s^2+6s+144)}$$

$$C(s)=\frac{5}{36}\frac{1}{s}-\frac{5}{36}\frac{s+6}{s^2+6s+144}$$

$$C(s)=\frac{5}{36}\frac{1}{s}-\frac{5}{36}\frac{(s+3)+\frac{3}{\sqrt{135}}\sqrt{135}}{(s+3)^2+135}$$

$$c(t)=\frac{5}{36}-\frac{5}{36}e^{-3t}\left(\cos\left[\sqrt{135}\right]t+\frac{3}{\sqrt{135}}\sin\left[\sqrt{135}\right]t\right)$$

f.

$$C(s)=\frac{s+5}{s(s+10)^2}$$

$$C(s)=\frac{1}{20}\frac{1}{s}-\frac{1}{20}\frac{1}{s+10}+\frac{1}{2}\frac{1}{(s+10)^2}$$

$$c(t)=\frac{1}{20}-\frac{1}{20}e^{-10t}+\frac{1}{2}te^{-10t}$$

Prob. 18:

a. N/A

b. $s^2+9s+18$, $\omega_n^2 = 18$, $2\zeta\omega_n = 9$, Therefore $\zeta = 1.06$, $\omega_n = 4.24$, overdamped.

d. $s^2+6s+144$, $\omega_n^2 = 144$, $2\zeta\omega_n = 6$, Therefore $\zeta = 0.25$, $\omega_n = 12$, underdamped.

f. $s^2+20s+100$, $\omega_n^2 = 100$, $2\zeta\omega_n = 20$, Therefore $\zeta = 1$, $\omega_n = 10$, critically damped.

Prob. 20:

a. $\omega_n^2 = 16$ r/s, $2\zeta\omega_n = 3$. Therefore $\zeta = 0.375$, $\omega_n = 4$. $T_s = \frac{4}{\zeta\omega_n} = 2.667$ s; $T_P = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}.} =$ 0.8472 s; %OS = $e^{-\zeta\pi / \sqrt{1-\zeta^2}}$ x 100 = 28.06 %; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1) = 1.4238$; therefore, $T_r = 0.356$ s.

Prob. 23:

a. $\zeta = \frac{-\ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%OS}{100})}} = 0.56$, $\omega_n = \frac{4}{\zeta T_s} = 11.92$. Therefore, poles $= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

$= -6.67 \pm j9.88$.

c. $\zeta\omega_n = \frac{4}{T_s} = 0.571$, $\omega_n\sqrt{1-\zeta^2} = \frac{\pi}{T_p} = 1.047$. Therefore, poles $= -0.571 \pm j1.047$.

Prob. 24:

The corresponding damping factor is $\xi = \frac{(-\ln(0.15)}{\sqrt{\pi^2 + ln^2(0.15)}} = 0.517$. The settling time is $T_s = \frac{4}{\xi\omega_n} = 0.7$ sec, so $\omega_n = 11.053$. The transfer function is

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{122.164}{s^2 + 11.43s + 122.164}.$$