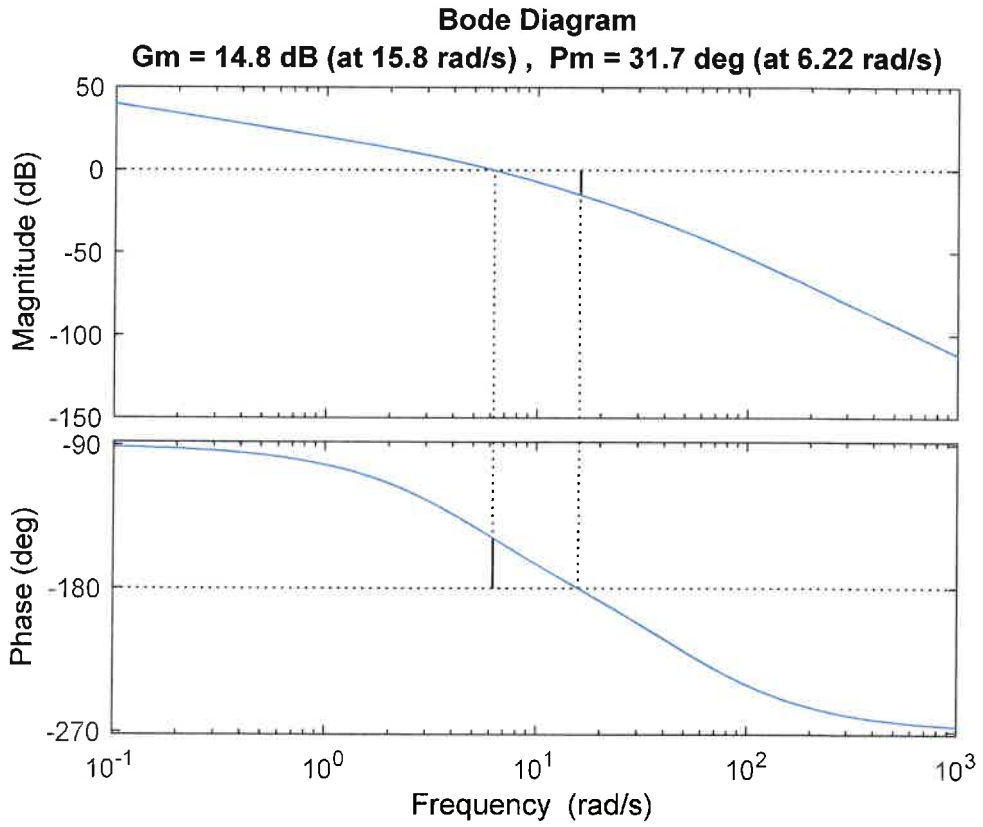
Example 1:

$$L(s) = \frac{2500}{s(s+5)(s+50)}$$

```
close all
s = tf('s')
g = 2500/(s*(s+5)*(s+50))
margin(g)
```



$$L(s) = \frac{2500}{s(s+5)(s+50)}$$

$$= \frac{10}{s} \cdot \frac{1}{1+\frac{s}{5}} \cdot \frac{1}{1+\frac{s}{50}}$$

UNITY GAIN FREQ. $\omega_g \Rightarrow |L(\omega_g)| = 1$

① FIRST SEGMENT $\forall f \leq 5$.

$\frac{10}{\omega_g} = 1 \Rightarrow \omega_g = 10.$ ✗

← NOT IN THE FREQ RANGE

② SECOND SEGMENT $5 \leq f \leq 50$.

$\frac{50}{\omega_g^2} = 1 \Rightarrow \omega_g = \sqrt{50} = 7.07$ ✓

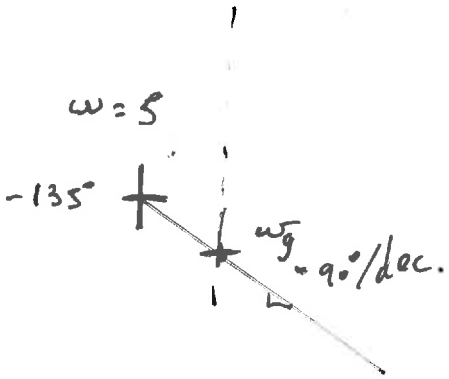
PHASE @ $\omega_g = 7.07$

FOR ANY ω → EXACT PHASE $= -90 - \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{\omega}{50}\right)$

PHASE @ $\omega = \omega_g = -90 - \tan^{-1}\left(\frac{7.07}{5}\right) - \tan^{-1}\left(\frac{7.07}{50}\right)$

$= -152.8°$

PHASE MARGIN $= 180 - 152.8 = 27.2°$



$$\angle @\ \omega_g = -135^\circ - 90 \times \log_{10}\left(\frac{\omega_g}{5}\right)$$

$$= -135 - 90 \log_{10}\left(\frac{7.07}{5}\right)$$

$$= -148.5^\circ$$

APPROX.

USE EXACT METHOD.

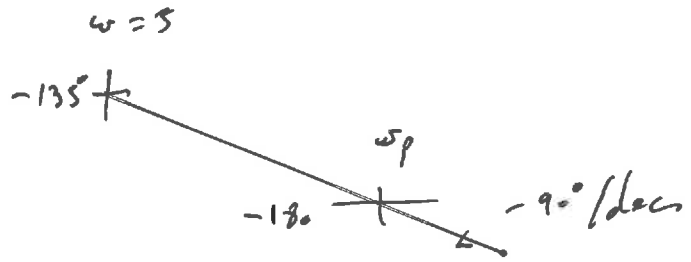
PHASE OF -180° OCCURS AT ω_p.

$$\text{EXACT PHASE} = -90 - \tan\left(\frac{\omega}{5}\right) - \tan\left(\frac{\omega}{50}\right)$$

$$\Rightarrow \quad -180 = -90 - \tan\left(\frac{\omega}{5}\right) - \tan\left(\frac{\omega}{50}\right).$$

?

APPROX. METHOD:



$$-180 = -135 - 90 \log_{10}\left(\frac{\omega_p}{5}\right)$$

$$\Rightarrow \quad 0.5 = \log_{10}\left(\frac{\omega_p}{5}\right)$$

$$\Rightarrow \quad \omega_1 = 5 \times 10^{0.5} = 5\sqrt{10}.$$

$$\omega_p = 15.8$$

MAG. @ $\omega_p = 15.8$: $\quad \frac{50}{\omega_p^2} = \frac{50}{(15.8)^2} = 0.2$

in dB $\quad 20 \log_{10}(0.2) = -13.97$ dB

$$\Rightarrow \quad GM = 13.97 \text{ dB}$$

SUMMARY (COMPARE WITH MATLAB RESULTS)

$PM = 27^\circ$ @ $\omega_g = 7.07$ $\quad GM = 13.97$ dB @ $\omega_p = 15.8$.

MATLAB: $PM = 31.7^\circ$ @ $\omega_g = 6.22$ $\quad GM = 14.8$ dB @ $\omega_p = 15.8$