

Exercises 15

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The below are in-class exercises designed to help solidify your understanding of the material covered in the notes. They will also aid you in completing some homework problems. Please work together with your group to complete as many of these problems as you can.

PN refers to the online textbook by Pishro-Nik available here. Please do not look at the solutions until after you have completed the problem or received hints from me.

Exercise 1

Customers arrive at a checkout lane at a rate of λ customers per second, modeled as a Poisson RP. Determine the probability that 5 customers will arrive during the first minute the lane is open and 5 will also arrive during the second minute it is open.

Exercise 2

A customer call service center opens at 9AM. The calls received follow a Poisson RP with $\lambda = 600$ calls/hour. The 20th call comes in at 9:01AM. When should we expect the next call to arrive?

Exercise 3

Packets arrive at a router according to a Poisson RP N_t with rate λ . Assume the service time for each packet is $T \stackrel{\text{i.i.d.}}{\sim} \exp(\beta)$ and is independent of N_t . What is the probability that k packets arrive during a service time?

Exercise 4

Let N_t be a Poisson RP with rate λ . Split N_t into two processes, $N_t^{(1)}$ and $N_t^{(2)}$, such that each event is randomly and independently assigned to $N_t^{(1)}$ with probability p and to $N_t^{(2)}$ otherwise. This implies $N_t = N_t^{(1)} + N_t^{(2)}$. Prove that $N_t^{(1)}$ is Poisson with rate $p\lambda$ and $N_t^{(2)}$ is Poisson with rate $(1-p)\lambda$.