Exercises 14

The below are in-class exercises designed to help solidify your understanding of the material covered in the notes. They will also aid you in completing some homework problems. Please work together with your group to complete as many of these problems as you can.

PN refers to the online textbook by Pishro-Nik available here. Please do not look at the solutions until after you have completed the problem or received hints from me.

Exercise 1

Explain why the below plots are not valid correlation functions of a WSS process.
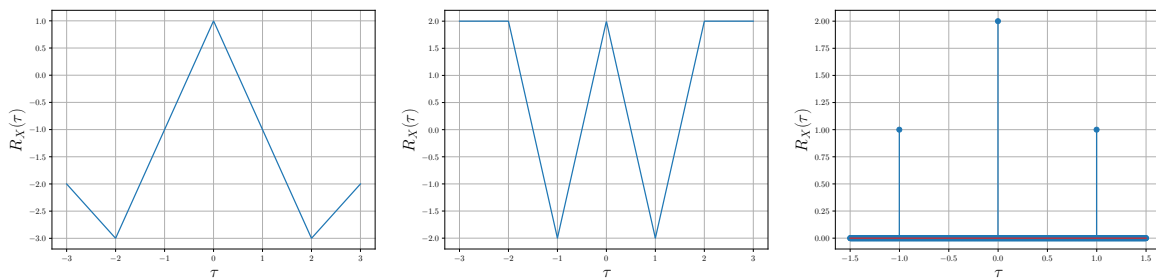
Exercise 2

Let X_t and Z_t be zero-mean, uncorrelated processes with power spectral densities

$$S_X(f) = \begin{cases} P/2, & |f| \leq B \\ 0, & \text{otherwise} \end{cases}$$

and

$$S_Z(f) = \frac{N}{2}, \quad \forall f.$$

Find the transfer function of the Wiener filter for the process $Y_t = X_t + Z_t$.

Exercise 3

A matched filter is to be found for the signal

$$v(t) = u(t) \left(e^{-ct} - e^{-\alpha ct} \right),$$

where $c, \alpha > 0$ and $u(t)$ denotes the unit step function

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}.$$

Let the noise have PSD

$$S_N(f) = \frac{c}{c^2 + (2\pi f)^2}.$$

Find the transfer function of the matched filter.

Exercise 4

A RP is called Gaussian if every finite sequence of elements from the process is a Gaussian RVec. A WSS Gaussian process X_t has the following properties

$$\mathbb{E}[X_t] = 0, \quad \forall t$$
$$R_X(t, s) = \cos(s - t), \quad \forall s, t.$$

(a) Explain why this is a valid correlation function.

(b) Find $S_X(f)$.

(c) Find P_X.

(d) Let $n(t)$ be a deterministic signal and define the RV X by

$$X = \int_0^T X_t n(t) dt,$$

where T is a constant. Show that the variance of X is

$$\sigma^2 = \int_0^T \int_0^T n(s)n(t) \cos(s - t) ds dt.$$