

Exercises 10

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The below are in-class exercises designed to help solidify your understanding of the material covered in the notes. They will also aid you in completing some homework problems. Please work together with your group to complete as many of these problems as you can.

PN refers to the online textbook by Pishro-Nik available here. Please do not look at the solutions until after you have completed the problem or received hints from me.

Exercise 1

PN 8.2.5, problem 4

Exercise 2

PN 9.1.10, problem 5

Exercise 3

Let X be a continuous RV with PDF

$$f_X(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Also, suppose that $Y | X = x \sim \text{Geometric}(x)$. Find the ML and MAP estimates of X given $Y = 5$.

Exercise 4

Let X and Y be jointly Gaussian RVs with mean zero, variances σ_X^2 and σ_Y^2 , and covariance σ_{XY} . We say their covariance matrix (see Lecture 11) is then

$$\mathbb{E} \begin{bmatrix} X \\ Y \end{bmatrix} \begin{bmatrix} X & Y \end{bmatrix} = \mathbb{E} \begin{bmatrix} X^2 & XY \\ XY & Y^2 \end{bmatrix} = \mathbb{E} \begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{bmatrix}.$$

Find $\mathbb{E}[X | Y]$, its PDF, the linear MMSE estimate, and the MAP estimate of X .

Hint: You can use the equation given in the book

$$f_{X|Y}(x | y) \sim \mathcal{N} \left(\frac{\sigma_X}{\sigma_Y} \rho_{XY} y, \sigma_X^2 (1 - \rho_{XY}^2) \right),$$

where

$$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

is the correlation coefficient.

Exercise 5

Let $Y_1, \dots, Y_N \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(\mu, \sigma^2)$. Find the ML estimates of μ and σ^2 .

Note: This is a more typical example of parameter estimation, also referred to as *classical* parameter estimation, where we want to find the parameters of a distribution.

Exercise 6

Let $Y_1, \dots, Y_N \stackrel{\text{i.i.d.}}{\sim} \text{Poisson}(\lambda)$. Find the ML estimate of the parameter λ .