

Exercises 8

Instructor Name: John Lipor

The below are in-class exercises designed to help solidify your understanding of the material covered in the notes. They will also aid you in completing some homework problems. Please work together with your group to complete as many of these problems as you can.

PN refers to the online textbook by Pishro-Nik available here. Please do not look at the solutions until after you have completed the problem or received hints from me.

Exercise 1

PN 5.2.5, problem 2

Exercise 2

PN 5.3.3, problem 1

Exercise 3

PN 5.4.0, problem 17

Exercise 4

PN 5.2.5, problem 3

Exercise 5

Toss a biased coin with $P(H) = p$ and let X denote the number of tosses until the first H appears. Find $\mathbb{E}[X]$ by conditioning on the outcome of the first coin toss.

Exercise 6

Let $X_1, X_2 \sim \mathcal{N}(0, 1)$. Define $R = g_1(X_1, X_2)$ and $\theta = g_2(X_1, X_2)$, where R is the length of the vector $[X_1 \ X_2]^T$ and θ is the angle this vector makes with the primary axis. Find $f_{R\theta}(r, \theta)$.

Exercise 7

Let $X, Y \sim f_{XY}$. Find $f_Z(z)$ if $Z = e^X Y$.

Hint: Do not try to apply the formula for two-dimensional transformations. Instead, note that you have two things that are random, so fixing one of them might make the problem easier. Try to use this trick to find the CDF of Z , then differentiate to find the PDF.

Exercise 8

Let $X_1, X_2 \stackrel{\text{i.i.d.}}{\sim} f_{X_1 X_2}(x_1, x_2) = f_{X_1}(x_1)f_{X_2}(x_2)$. Define $Z_1 = \min(X_1, X_2)$ and $Z_2 = \max(X_1, X_2)$. Find $f_{Z_1 Z_2}(z_1, z_2)$.