Exercises 6

The below are in-class exercises designed to help solidify your understanding of the material covered in the notes. They will also aid you in completing some homework problems. Please work together with your group to complete as many of these problems as you can.

PN refers to the online textbook by Pishro-Nik available here. Please do not look at the solutions until after you have completed the problem or received hints from me.

Exercise 1

PN 4.2.6, problem 3

Exercise 2

PN 4.4.1, problem 1

Exercise 3

PN 4.4.1, problem 3

Exercise 4

PN 4.4.1, problem 4

Exercise 5

PN 4.4.1, problem 6

Exercise 6

PN 4.4.1, problem 16

Exercise 7

Let $X_1, X_2, \ldots, X_n \sim \mathcal{N}(0, \sigma^2)$ but not necessarily independent. Let $Y = \max\{X_1, \ldots, X_n\}$. Show that

$$\mathbb{E}[Y] \leq \sigma\sqrt{2\log n}.$$

Exercise 8

Let the RV N denote the number of stars in a region of space with volume V. Assume $N \sim \text{Poisson}(\rho V)$, where ρ is the density of stars in space. Choose an arbitrary point in space, and let X denote the distance from that point to the nearest star. Find $f_X(x)$ in terms of ρ.

Exercise 9

You arrive at a bank with two tellers whose service times are independent RVs $X_1 \sim \exp(\lambda_1)$ and $X_2 \sim \exp(\lambda_2)$, respectively. You are first in line to be served. What is the probability you are served by teller 1?