

Exercises 5

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The below are in-class exercises designed to help solidify your understanding of the material covered in the notes. They will also aid you in completing some homework problems. Please work together with your group to complete as many of these problems as you can.

PN refers to the online textbook by Pishro-Nik available here. Please do not look at the solutions until after you have completed the problem or received hints from me.

Exercise 1

PN 4.1.4, problem 1

Exercise 2

PN 4.2.5, problem 1

Exercise 3

Let $X \sim \mathcal{N}(2, 4)$ and define $Y = 3 - 2X$. Find $\mathbb{E}[Y]$, $\text{var}(Y)$, and $\text{cov}(X, Y)$.

Exercise 4

PN 4.2.5, problem 5. **Hint:** Use the fact that $\int_0^\infty te^{-t^2/2} dt = \sqrt{\pi/2}$.

Exercise 5

Suppose $X \sim \text{Unif}([1, 2])$, and given $X = x$, Y is exponential with parameter $\lambda = x$. Find $\text{cov}(X, Y)$.

Exercise 6

PN 4.1.4, problem 3

Exercise 7

(a) Let X be a non-negative RV. Show that

$$\mathbb{E}[X] = \int_0^\infty P(X > t) dt.$$

(b) Let the CDF of X be

$$F_X(x) = P(X \leq x) = 1 - e^{-x^2}, \quad x \geq 0.$$

Find $\mathbb{E}[X]$.

Exercise 8

A company produces independent voltage regulators whose outputs are $\exp(\lambda)$ RVs. In a batch of 10 regulators, find the probability that exactly 3 produce outputs greater than v volts.

Exercise 9

Assume we transmit a signal at m volts, which is then corrupted by additive $\mathcal{N}(0, \sigma^2)$ noise. Compute the likelihood ratio for this signal.