EE 520: Random Processes **Fall 2021**

Exercises 4

Instructor Name: John Lipor

The below are in-class exercises designed to help solidify your understanding of the material covered in the notes. They will also aid you in completing some homework problems. Please work together with your group to complete as many of these problems as you can.

PN refers to the online textbook by Pishro-Nik available here. Please do not look at the solutions until after you have completed the problem or received hints from me.

Exercise 1

Let X and Y be two RVs and let g and h be two functions. Show that

$$\mathbb{E}[g(X)h(Y) \mid X] = g(X)\mathbb{E}[h(Y) \mid X].$$

Exercise 2

Prove the law of total variance, i.e.,

$$\operatorname{var}(X) = \mathbb{E}[\operatorname{var}(X \mid Y)] + \operatorname{var}\left(\mathbb{E}[X \mid Y]\right).$$

Hint: Let $V = \operatorname{var}(X \mid Y)$ and $Z = \mathbb{E}[X \mid Y]$. Write out V in terms of X, Y, and Z, then find $\mathbb{E}[V]$. Next find $\operatorname{var}(Z)$.

Exercise 3

Let N be the number of customers that visit a certain store on a given day. Suppose that we know $\mathbb{E}[N]$ and $\operatorname{var}(N)$. Let X_i be the amount that the ith customer spends on average. We assume the X_i's are independent of each other and of N. We further assume that they have the same mean and variance. Let Y be the store's total sales, i.e.,

$$Y = \sum_{i=1}^{N} X_i.$$

Find $\mathbb{E}[Y]$.

Exercise 4

Suppose $X \sim \text{Unif}([1, 2])$, and given $X = x$, Y is exponential with parameter $\lambda = x$. Find $\operatorname{cov}(X, Y)$.

Exercise 5

PN 5.3.3, problem 2

Exercise 6

Use the Cauchy-Schwarz inequality to show that for any two RVs X and Y

$$|\rho(X, Y)| \leq 1.$$

Also, $|\rho(X, Y)| = 1$ if and only if $Y = aX + b$ for some constants $a, b \in \mathbb{R}$.

Exercise 7

Let $X \sim \text{Poisson}(\lambda)$. Find $P(X > 1)$.

Exercise 8

A class has 15 students, each having probability $p = 0.1$ of getting an "A" in the course. Assuming the grades are independent, find the probability that exactly one student gets an "A."

Exercise 9

A hen lays N eggs, where $N \sim \text{Poisson}(\lambda)$. Each egg hatches with probability p independently of the other eggs. Let K be the number of resulting chicks. Find $\mathbb{E}[K \mid N]$, $\mathbb{E}[K]$, and $\mathbb{E}[N \mid K]$.

Exercise 10

A building has n floors. Suppose M people enter at floor 1, where $M \sim \text{Poisson}(10)$. Each person is equally likely to exit at any floor $2, \ldots, n$. Find the expected number of floors on which at least one person exits the elevator.