Ex 1-3: See PN.

Ex 4:

$$\mathbb{E}[Y^2] = \mathbb{E}[(X_5 - X_2)^2]$$

$$= \mathbb{E}[X_5^2 + X_2^2 - 2X_5X_2]$$

$$= 2R_x(0) - 2R_x(3)$$

$$= 2A(1 - e^{3\alpha})$$

Ex 5

For X_t to be WSS, we need its mean to be independent of time and its correlation to be a function only of the time difference.

$$m_x(t) = \mathbb{E}[X_t]$$

$$= \mathbb{E}[A \cos(\omega t + \theta)]$$

$$= \cos(\omega t + \theta)\,\mathbb{E}[A]$$

So $m_x(t)$ is only independent of time if $\mathbb{E}[A] = 0$.

$$R_x(t, s) = \mathbb{E}[X_t X_s]$$

$$= \mathbb{E}[A^2 \cos(\omega t + \theta) \cos(\omega s + \theta)]$$

$$= \mathbb{E}[A^2] \cos(\omega t + \theta) \cos(\omega s + \theta)$$

$$= \mathbb{E}[A^2]\, \tfrac{1}{2} \big(\cos(\omega(t+s) + 2\theta) + \cos(\omega(t-s)) \big)$$

which depends on t and s, so X_t is <u>not</u> WSS.

Ex 6

First find $S_{XY}(f)$, then take the inverse FT. Please feel free to use a FT table of your choice for these!

$$S_{XY}(f) = H^*(f)\, S_X(f)$$

Using a table, we see that

$$S_X(f) = \sqrt{2\pi}\, e^{-(2\pi f)^2/2}$$

Note that $j2\pi f$ is the FT of a <u>differentiator</u> so taking the inverse FT

$$R_{XY}(\tau) = -\frac{d}{d\tau} R_X(\tau)$$

$$= \tau e^{-\tau^2/2}$$

We can apply the same idea to get $R_Y(\tau)$ via $S_Y(f)$.

$$S_Y(f) = |H(f)|^2 S_X(f)$$

$$= (-j\,2\pi f)(j\,2\pi f)\, S_X(f)$$

$$\Rightarrow R_Y(\tau) = -\frac{d^2}{d\tau^2} R_X(\tau)$$

$$= \frac{d}{d\tau} R_{XY}(\tau)$$

$$= (1-\tau^2)\, e^{-\tau^2/2}$$