## Ex 1-8: See PN.

## Ex 9

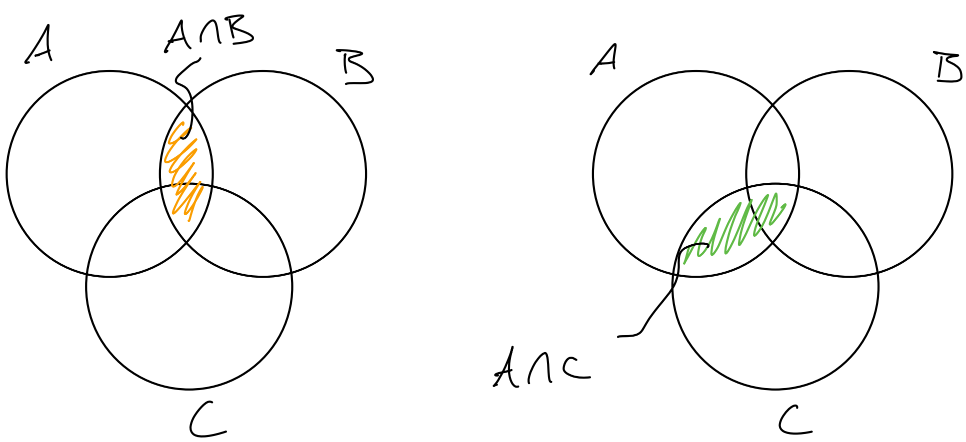
a) The set is everything in $A$ or $B$ but not in their intersection so

$$S = (A \cup B) \setminus (A \cap B)$$

b) The set includes the parts of $A \cap B$ that are <u>not</u> in $C$

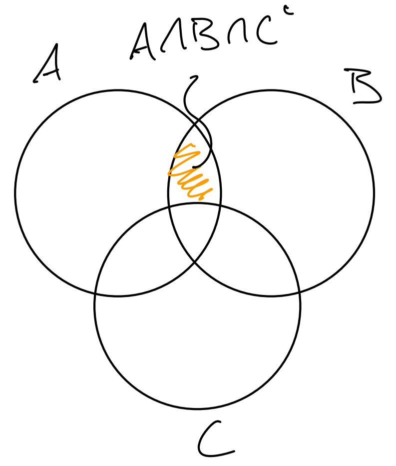
$$S = (A \cap B) \setminus C.$$

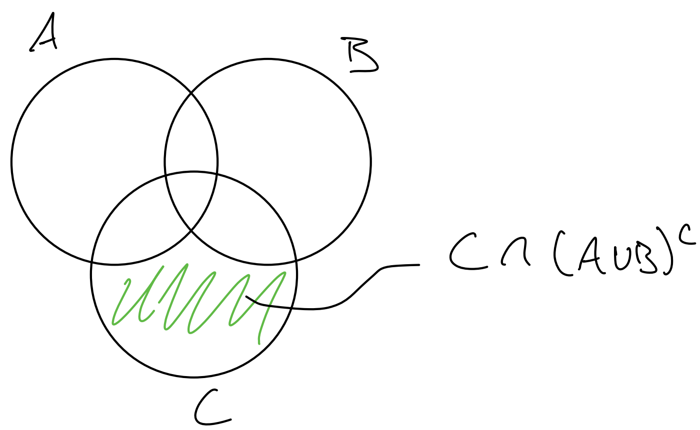
c) Breaking the shaded region up, we have




So $\quad S = (A \cap B) \cup (A \cap C)$

d) Breaking the sets up, we have




So $\quad S = (A \cap B \cap C^c) \cup (C \cap (A \cup B)^c)$

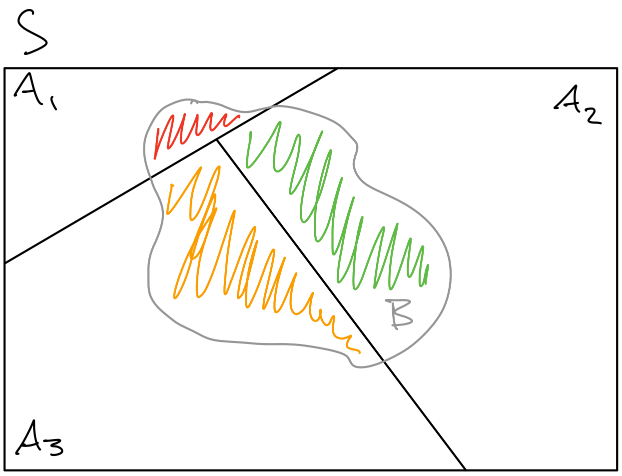
Many of these can be further simplified using set algebra, but this is typically done only if we know certain probabilities (e.g., if we know $P(A \cap B^c)$).

## Ex 10

By the definition of a partition, we have

$$S = A_1 \cup A_2 \cup A_3 \quad \text{and} \quad \begin{array}{l} A_1 \cap A_2 = \phi \\ A_1 \cap A_3 = \phi \\ A_2 \cap A_3 = \phi \end{array} .$$

The following diagram may be useful:




$|B \cap A_1| = 10$

$|B \cap A_2| = 20$

$|B \cap A_3| = 15$

Since $A_1, A_2, A_3$ are mutually disjoint, we have that

$$|B| = \sum_{i=1}^{3} |A \cap B_i| = 10 + 20 + 15 = 45.$$

## Ex 11

Note that a union corresponds to a "for some / any" statement, so we have

$$\bigcup_{n=1}^{\infty} \left[0, \frac{n-1}{n}\right) = \left\{ x \in \mathbb{R} : 0 \le x < \frac{n-1}{n} \text{ for some } n \in \mathbb{N} \right\}$$

Since $n-1 < n$, this set must not contain anything greater than or equal to 1. However, for any value $x < 1$, we can find a large enough $n$ so that $\frac{n-1}{n} > x$ (note that $\lim_{n \to \infty} \frac{n-1}{n} = 1$). Therefore

$$\bigcup_{n=1}^{\infty} \left[0, \frac{n-1}{n}\right) = [0, 1).$$

## Ex 12

Note that an intersection corresponds to a "for all" statement. So

$$\bigcap_{n=1}^{\infty} [0, \tfrac{1}{n}) = \{ x \in \mathbb{R} : x < \tfrac{1}{n} \ \forall n \in \mathbb{N} \} = \{0\}$$

Clearly 0 is in this set. Why not anything else?

Take some small value $\varepsilon > 0$. Then there exists an $n$ such that $\frac{1}{n} < \varepsilon$, meaning $\varepsilon$ cannot be in the set for any $\varepsilon > 0$.

## Ex 13

$A \cap B \in \mathcal{F}$ : Note $A \cap B = (A^c \cup B^c)^c$.

1) $A \in \mathcal{F} \Rightarrow A^c \in \mathcal{F}$

2) $B \in \mathcal{F} \Rightarrow B^c \in \mathcal{F}$

3) $A^c, B^c \in \mathcal{F} \Rightarrow A^c \cup B^c \in \mathcal{F}$

4) $A^c \cup B^c \in \mathcal{F} \Rightarrow (A^c \cup B^c)^c \in \mathcal{F}$

$A \setminus B \in \mathcal{F}$ : Note $A \setminus B = A \cap B^c$

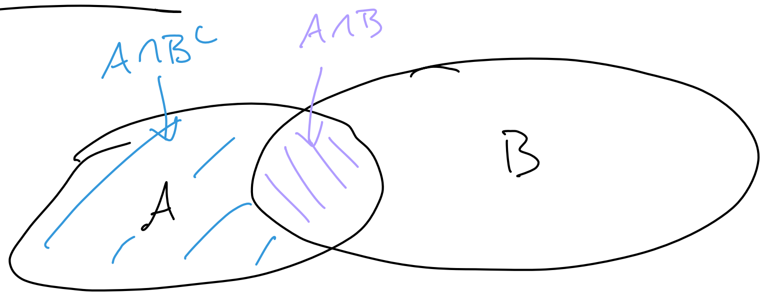
1) $B \in \mathcal{F} \Rightarrow B^c \in \mathcal{F}$

2) $A, B^c \in \mathcal{F} \Rightarrow A \cap B^c \in \mathcal{F}$ (by above)

$\underline{A \Delta B \in \mathcal{F}}$: Follows directly by $A \setminus B \in \mathcal{F}$ and fact that $\mathcal{F}$ is closed under countable unions.

## Ex 14

$\underline{P(A \cap B^c)}$ : Note that $P(A) = P(A \cap B) + P(A \cap B^c)$




The events $A \cap B$ and $A \cap B^c$ are disjoint, so

$$P(A) = P(A \cap B) + P(A \cap B^c).$$

Thus

$$\begin{aligned} P(A \cap B^c) &= P(A) - P(A \cap B) \\ &= P(A) - \big(P(A) + P(B) - P(A \cup B)\big) \\ &= P(A \cup B) - P(B) \end{aligned}$$

$\underline{P(B \cup (A \cap B^c))}$ :

$$\begin{aligned} P(B \cup (A \cap B^c)) &= P((B \cup A) \cap (B \cup B^c)) \\ &= P((B \cup A) \cap \Omega) \\ &= P(A \cup B) \end{aligned}$$

## Ex 15

First note that $P(A^c) \leq \delta$ and $P(B^c) \leq \delta$, since $P(A) = 1 - P(A^c)$. By the union bound,

$$P(A^c \cup B^c) \leq 2\delta.$$

Now apply DeMorgan's law to see that

$$(A^c \cup B^c)^c = A \cap B$$

and therefore

$$P\left((A^c \cup B^c)^c\right) = 1 - P(A^c \cup B^c)$$

$$\geq 1 - 2\delta.$$