EE 510: Mathematical Foundations of Machine Learning **Winter 2021**

Exercises 3

Pages: 2.1 - 2.29 *Instructor Name: John Lipor*

Exercise 1

Let

$$A = \begin{bmatrix} 4 & 6 \\ 6 & 9 \end{bmatrix},$$

which has eigenvalues 0 and 13. Is $v = \begin{bmatrix} 2 & 3 \end{bmatrix}^T$ a valid eigenvector for A? If not, provide a valid eigenvector.

Exercise 2

- Show that if the columns of $V \in \mathbb{R}^{n \times n}$ are orthogonal, then $V^T V$ is a diagonal matrix.
- What are the values of the diagonal elements of $V^T V$?
- What must be true of the columns of V such that $V^T V = I$?

Exercise 3

Let $X \in \mathbb{R}^{m \times n}$. Prove that the Gram matrix $X^T X$ and outer product matrix $X X^T$ are both symmetric.

Exercise 4

Verify that the permutation matrix P on pg. 2.6 is normal, i.e., that $P^T P = P P^T$.

Exercise 5

For the rotation matrix

$$V = \begin{bmatrix} \cos\theta & -q\sin\theta \\ \sin\theta & q\cos\theta \end{bmatrix},$$

verify that $V^T V = I$, where $q \in \{\pm 1\}$.

Exercise 6

Let $A \in \mathbb{F}^{m \times n}$ (i.e., A may have real or complex values), and let $y = v_1 + v_2$, where v_1, v_2 are the *right* singular vectors of A associated with the largest two singular values. What is $\|Ay\|$?

Exercise 7

For $A \in \mathbb{F}^{m \times n}$, what vectors $x, z \in \mathbb{F}^m$ make $A^T x$ and $A^T z$ orthogonal?

Exercise 8

Work through the matrix 2-norm proof on pg. 2.20 carefully. Note and discuss any steps that you do not understand.

Exercise 9

Work through the EVD/SVD example on pg. 2.24. Verify that each line is indeed a valid EVD/SVD.

Exercise 10

If $A = X^T X$ for $X \in \mathbb{R}^{m \times n}$, what are the eigenvectors and eigenvalues of A in terms of the singular vectors and singular values of X?