EE 510: Mathematical Foundations of Machine Learning **Winter 2021**

Exercises 2

Pages: 1.21 - 1.61 *Instructor Name: John Lipor*

Problems denoted by "BV X.YZ" are exercises from the book *Introduction to Applied Linear Algebra* by Boyd and Vandenberghe, which can be downloaded for free at the authors' website here.

Exercise 1

For $x, y \in \mathbb{R}^n$, the inner product/dot product is defined as $x^T y = \langle x, y \rangle = \sum_{i=1}^n x_i y_i$. Show that $x^T y = y^T x$.

Exercise 2

Show that the inner product is linear in the first argument, i.e., show that $\langle x + y, z \rangle = \langle x, z \rangle + \langle y, z \rangle$ and $\langle \alpha x, y \rangle = \alpha \langle x, y \rangle$, where $\alpha \in \mathbb{R}$.

Exercise 3

BV 1.11

Exercise 4

BV 1.16

Exercise 5

Let $A \in \mathbb{R}^{20 \times 37}$. What size is $A_{17,:}$? $A_{:,3}$? Let $B \in \mathbb{R}^{37 \times 4}$. What size is AB?

Exercise 6

Let $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times p}$, $C \in \mathbb{R}^{n \times q}$.

- What must be true for $A(B + C)$ to be a valid operation?
- What is $A(B + C)$ when $C = 0$? When $A = 0$?

Exercise 7

Let $A \in \mathbb{R}^{3 \times 127}$, $C \in \mathbb{R}^{4000 \times 1}$, $B \in \mathbb{R}^{127 \times 4000}$. What size is ABC? What is the most memory-efficient way to place parentheses when computing this product?

Exercise 8

Where is the associative property used on the Exercises 1 sheet?

Exercise 9

Let $A \in \mathbb{R}^{m \times n}$ and $v \in \mathbb{R}^n$. Where must parentheses be placed to make $Av^T v$ a valid operation?

Exercise 10

Let $A \in \mathbb{R}^{200\times 40}$. What size must I be to compute IA? To compute AI?

Exercise 11

Let

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 2 \\ 5 & 7 \end{bmatrix}.$$

What is AB? What is BA? What can you conclude about the relationship between AB and BA? Can you construct an example where they are the same?

Exercise 12

If $x, y \in \mathbb{R}^n$ have angle $0°$ between them, what can you say about $\|x + y\|$? If $x, y \in \mathbb{R}^n$ are orthogonal, what can you say about $\|x + y\|^2$?

Exercise 13

For $x, y \in \mathbb{R}^n$, verify that

- $(x + y)^T(x - y) = \|x\|^2 - \|y\|^2$
- $\|x + y\|^2 + \|x - y\|^2 = 2\left(\|x\|^2 + \|y\|^2\right)$.

Exercise 14

For $x \in \mathbb{R}^n$, show that $\|x\|^2 = \mathrm{tr}(xx^T)$.